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A structural constitutive model for the strain rate-dependent behavior of anterior cruciate ligaments

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Abstract

A structural continuum constitutive law is formulated to illustrate the anterior cruciate ligament (ACL) mechanical behavior. The constitutive law is obtained by modifying previously proposed structural models for soft tissues to include the description of strain rate-dependent effects. The ligamentous tissue anisotropy, non-linearity, incompressibility, and strain rate related properties are taken into account. The collagen fibers, which comprise the ACL, are assumed to be the only load-bearing component of the tissue. They are oriented in different directions, undulated in the stress-free configuration and they gradually become taut upon loading. Moreover, the taut collagen fibers are characterized by a Kelvin–Voigt-type viscoelastic behavior. The fiber spatial orientation and gradual recruitment are represented statistically by probability density functions. Published experimental stress–strain data of the ACL-bone complex are used to assess the constitutive model. The model is tested by assuming that the ACL has a perfectly parallel collagen structure and undergoes an isochoric, axisymmetric deformation. Furthermore, the fiber recruitment is defined by a one-sided probability density function. The structural parameters are able to fit the stress–strain curves at different strain rates.

Keywords: ACL; Strain rate effects; Structural model

1. Introduction

The anterior cruciate ligament (ACL) is the most commonly injured of the major knee joint ligaments. Epidemiological research has indicated that approximately 1 in 3000 individuals experiences ACL injury every year (McNair et al., 1990). Because of the high incidence of ACL injuries, the mechanical factors, which play important roles in the mechanisms of injuries, need to be accurately investigated. It has been

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estimated that injuries occur at strain rates that range between 50%/s and 150,000%/s (Crowninshield and Pope, 1976). Hence, in order to understand the mechanisms of injuries, studying the mechanical features of the ACL at different strain rates is crucial.

Phenomenological and structural constitutive relations have been previously adopted to simulate the ACL mechanical response. However, only few phenomenological models have been proposed to describe the strain rate-dependent mechanical behavior of the ACL (Pioletti et al., 1998; Limbert and Middleton, 2004). Unlike phenomenological models, structural constitutive laws are formulated by taking into account the tissue's morphology. Consequently, the material parameters embodied in these models are directly related to the tissue's structure.

Several structural models have been proposed to describe various connective tissues (Lanir, 1979, 1980; Kastelic et al., 1980; Decraemer et al., 1980; Lanir, 1983; Horowitz et al., 1988; Kwan and Woo, 1989; Zioupos and Barbenel, 1994; Hurschler et al., 1997; Wren and Carter, 1998; Liao and Belkoff, 1999; Sacks, 2000; Thornton et al., 2001). Some assumptions regarding the tissues' behavior are common in these models. The fibers, which constitute the tissues, appear to have a crimped morphology in the unstressed state. When loaded, they lose their waviness and align in the direction of tension. It is believed that the sequential uncrimping of the fibers gives rise to the characteristic toe region in the stress–strain relationship. The fiber mechanical response is typically assumed to be linear elastic or, rarely, linear viscoelastic (Lanir, 1980; Decraemer et al., 1980; Thornton et al., 2001). When the fibers are all straight, the overall mechanical response of the tissue becomes linear.

The mechanical behavior of the ligamentous tissue is determined by its constituents, their structures, and their interactions. The ligament is primarily composed of a densely organized network of collagen fibers, with a small amount of elastin fibers. All the fibers are embedded in a matrix of proteoglycans (Viidik, 1990). Although there is a paucity of information on the mechanical function of each component of the ligament, collagen fibers appear to have the predominant mechanical role in collagenous tissues (Minnis et al., 1973).

The objective of this study is to develop a constitutive model to accurately describe the mechanical behavior of ACL based on the collagen fibrous structure. By taking into account the orientation, the crimping, and the viscoelastic component of collagen fibers, the model accounts for the effect of strain rate on the mechanical response of the ACL. Available experimental data from the literature (Danto and Woo, 1993; Pioletti et al., 1999) are used to assess the model.

2. Model formulation

An incompressible, three-dimensional constitutive law is proposed that is based on the collagenous fibrous structure of most ligaments. It is a modification of structural constitutive relations for soft tissues, originally proposed by Lanir (1979) and Lanir (1983), that incorporates strain rate effects. In the following formulation, the fibrous network is comprised of variously undulated fibers oriented in different directions. Collagen fibers are undulated in the slack configuration and unable to support load. They are gradually straightened under strain, at which point they manifest a viscoelastic behavior. Both the spatial arrangement and the waviness of the collagen fibers are defined stochastically.

2.1. Constitutive equation

The existence of elastic and viscous potentials, $W_{\mathrm{e}}(\boldsymbol{C})$ and $W_{\mathrm{v}}(\boldsymbol{C},\dot{\boldsymbol{C}})$, respectively, is assumed such that the first Piola–Kirchhoff stress tensor $\boldsymbol{P}$ can be expressed as

$$\boldsymbol{P} = -p\boldsymbol{F}^{-\mathsf{T}} + 2\boldsymbol{F}\cdot\left(\frac{\partial W_{\mathrm{e}}(\boldsymbol{C})}{\partial \boldsymbol{C}} + \frac{\partial W_{\mathrm{v}}(\boldsymbol{C},\dot{\boldsymbol{C}})}{\partial \dot{\boldsymbol{C}}}\right), \tag{1}$$

where p is an indeterminate pressure enforcing the incompressibility assumption, $\boldsymbol{F}$ is the deformation gradient tensor, $\boldsymbol{F}^{\mathrm{T}}$ and $\boldsymbol{F}^{-\mathrm{T}}$ are its transpose and inverse transpose, respectively, $\boldsymbol{C} \equiv \boldsymbol{F}^{\mathrm{T}} \cdot \boldsymbol{F}$ is the right Cauchy–Green deformation tensor, and $\dot{\boldsymbol{C}}$ is its material time derivative (Noll, 1958; Pioletti et al., 1998). A "dot product" notation is used, wherein a vector $\boldsymbol{u}$ is mapped by a second-order tensor $\boldsymbol{A}$ into the vector $\boldsymbol{A} \cdot \boldsymbol{u}$ and the composition of two second-order tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ is another second-order tensor denoted by $\boldsymbol{A} \cdot \boldsymbol{B}$, so that $(\boldsymbol{A} \cdot \boldsymbol{B}) \cdot \boldsymbol{u} = \boldsymbol{A} \cdot (\boldsymbol{B} \cdot \boldsymbol{u})$ for an arbitrary vector $\boldsymbol{u}$. The last expression thus admits the unambiguous representation $\boldsymbol{A} \cdot \boldsymbol{B} \cdot \boldsymbol{u}$. The tensor product of two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ is a second-order tensor denoted by $\boldsymbol{uv}$ and defined such that $\boldsymbol{uv} \cdot \boldsymbol{a} = (\boldsymbol{uv}) \cdot \boldsymbol{a} = \boldsymbol{u}(\boldsymbol{v} \cdot \boldsymbol{a})$ for an arbitrary vector $\boldsymbol{a}$.

In the absence of the viscous potential, (1) yields the usual dissipation-free, incompressible hyperelastic response. Thus, the viscous potential $W_{\mathrm{v}}(\boldsymbol{C}, \dot{\boldsymbol{C}})$ accounts for dissipation (Pioletti et al., 1998). Sufficient conditions for satisfaction of the Clausius–Duhem inequality are that the viscous potential be continuous, non-negative, and convex and that $W_{\mathrm{v}}(\boldsymbol{C}, 0) = 0$.

Let $R(\boldsymbol{m})$ be the probability density for collagen fibers whose mean axes in the reference configuration are parallel to the unit vector $\boldsymbol{m}$. Both the elastic and the viscous potentials are assumed to be determined solely by the collagen fibers' extension—shear and bending energies are not taken into account. Accordingly, it is assumed that the elastic and viscous potentials can be represented as

$$W_{\mathrm{e}}(\boldsymbol{C}) = \int_{\Sigma} R(\boldsymbol{m}) w_{\mathrm{e}}(\Lambda(\boldsymbol{C}, \boldsymbol{m}))\, \mathrm{d}\Sigma, \tag{2}$$

$$W_{\mathrm{v}}(\boldsymbol{C}, \dot{\boldsymbol{C}}) = \int_{\Sigma} R(\boldsymbol{m}) w_{\mathrm{v}}(\Lambda(\boldsymbol{C}, \boldsymbol{m}), \dot{\Lambda}(\boldsymbol{C}, \dot{\boldsymbol{C}}, \boldsymbol{m}))\, \mathrm{d}\Sigma, \tag{3}$$

where Σ is the set of all material directions and $w_{\mathrm{e}}(\Lambda)$ and $w_{\mathrm{v}}(\Lambda, \dot{\Lambda})$ are the collagen fiber elastic and viscous potentials corresponding to axial fiber stretch Λ and stretch rate $\dot{\Lambda}$, given by (Ogden, 1997)

$$\Lambda(\boldsymbol{C}, \boldsymbol{m}) = \sqrt{\boldsymbol{m} \cdot \boldsymbol{C} \cdot \boldsymbol{m}}, \quad \dot{\Lambda}(\boldsymbol{C}, \dot{\boldsymbol{C}}, \boldsymbol{m}) = \frac{1}{2} \frac{\boldsymbol{m} \cdot \dot{\boldsymbol{C}} \cdot \boldsymbol{m}}{\sqrt{\boldsymbol{m} \cdot \boldsymbol{C} \cdot \boldsymbol{m}}}. \tag{4}$$

It needs to be emphasized that the elastic and viscous potentials of the ligamentous tissues are defined by summing the contributions of the collagen fibers that are aligned along different directions $\boldsymbol{m}$ according to the probability density $R(\boldsymbol{m})$. Therefore, no specific assumption on the material symmetry of these tissues is required.

Introducing the fiber elastic stress $\sigma_{\mathrm{e}}(\Lambda)$ and the fiber viscous stress $\sigma_{\mathrm{v}}(\Lambda, \dot{\Lambda})$ as follows

$$\sigma_{\mathrm{e}}(\Lambda) \equiv \frac{\mathrm{d}w_{\mathrm{e}}(\Lambda)}{\mathrm{d}\Lambda}, \quad \sigma_{\mathrm{v}}(\Lambda, \dot{\Lambda}) \equiv \frac{\partial w_{\mathrm{v}}(\Lambda, \dot{\Lambda})}{\partial \dot{\Lambda}}, \tag{5}$$

and noting that

$$\frac{\partial w_{\mathrm{e}}}{\partial \boldsymbol{C}} = \frac{\mathrm{d}w_{\mathrm{e}}}{\mathrm{d}\Lambda} \frac{\partial \Lambda}{\partial \boldsymbol{C}}, \quad \frac{\partial w_{\mathrm{v}}}{\partial \dot{\boldsymbol{C}}} = \frac{\partial w_{\mathrm{v}}}{\partial \dot{\Lambda}} \frac{\partial \dot{\Lambda}}{\partial \dot{\boldsymbol{C}}}, \tag{6}$$

where

$$\frac{\partial \Lambda}{\partial \boldsymbol{C}} = \frac{\partial \dot{\Lambda}}{\partial \dot{\boldsymbol{C}}} = \frac{\boldsymbol{mm}}{\Lambda(\boldsymbol{C}, \boldsymbol{m})}, \tag{7}$$

the constituive Eq. (1) then assumes the form

$$\boldsymbol{P} = -p\boldsymbol{F}^{-\mathrm{T}} + \boldsymbol{F} \cdot \int_{\Sigma} R(\boldsymbol{m}) \frac{\boldsymbol{mm}}{\Lambda(\boldsymbol{C}, \boldsymbol{m})} \left[\sigma_{\mathrm{e}}(\Lambda(\boldsymbol{C}, \boldsymbol{m})) + \sigma_{\mathrm{v}}(\Lambda(\boldsymbol{C}, \boldsymbol{m}), \dot{\Lambda}(\boldsymbol{C}, \dot{\boldsymbol{C}}, \boldsymbol{m}))\right] \mathrm{d}\Sigma. \tag{8}$$

Once the collagen fiber orientation distribution $R(\boldsymbol{m})$ and axial constitutive relations $\sigma_{\mathrm{e}}(\Lambda)$ and $\sigma_{\mathrm{v}}(\Lambda, \dot{\Lambda})$ have been specified, relation (8) can be employed to predict the ligament's strain-dependent behavior.

2.2. Recruitment model

The fiber recruitment model, which is characterized statistically by a probability density function for the stretch necessary to straighten a crimped fiber, has been employed by a number of researchers in biomechanics (Lanir, 1979, 1980; Decraemer et al., 1980; Horowitz et al., 1988; Lanir, 1983; Hurschler et al., 1997; Zioupos and Barbenel, 1994; Sacks, 2000). The novelty of the proposed structural model is in the introduction of collagen fibers' viscous effects to describe the ligament's strain rate effect.

Collagen fibers are undulated, or crimped, in the stress-free configuration. They are assumed to support load only after becoming taut—the load necessary to straighten the fibers is assumed to be negligible in comparison. Thus, the fiber elastic and viscous stresses are given by

$$\sigma_e(\Lambda) = \int_1^{\Lambda} g(\Lambda_s)\hat{\sigma}_e\left(\frac{\Lambda}{\Lambda_s}\right) d\Lambda_s, \tag{9}$$

$$\sigma_v(\Lambda, \dot{\Lambda}) = \int_1^{\Lambda} g(\Lambda_s)\hat{\sigma}_v\left(\frac{\Lambda}{\Lambda_s}, \frac{\dot{\Lambda}}{\Lambda_s}\right) d\Lambda_s, \tag{10}$$

where $g(\Lambda_s)$ is the probability density for fibers which become taut at a stretch Λ_s and Λ/Λ_s is the stretch with respect to the fiber's taut configuration. The probability density must satisfy the normality condition $\int_1^{\infty} g(\Lambda) d\Lambda = 1$. The values of $\hat{\sigma}_e(\Lambda_t)$ and $\hat{\sigma}_v(\Lambda_t, \dot{\Lambda}_t)$ represent the elastic and viscous stresses for a taut fiber stretched an amount $\Lambda_t = \Lambda/\Lambda_s$ and with a stretch rate of $\dot{\Lambda}_t = \dot{\Lambda}/\Lambda_s$. They are subject to the constraints $\hat{\sigma}_e(1) = 0$ and $\hat{\sigma}_v(\Lambda, 0) = 0$. It is worth noting that one could postulate fiber recruitment relations, analogous to (9) and (10), in terms of the fiber elastic potential $w_e(\Lambda)$ (Lanir, 1979, 1983) and the fiber viscous potential $w_v(\Lambda, \dot{\Lambda})$.

2.3. Fiber elastic and viscous stresses

The structural model presented above correlates the gross mechanical response of the ACL to the collagen fibers' mechanical response. Based on studies by Sasaki and Odajima (1996), each fiber is assumed to have a Kelvin–Voight-type viscoelastic constitutive behavior.

There is often some ambiguity in reported experimental results regarding the precise definition of the measured strains. Assuming that the strain measured is the logarithmic strain, $\varepsilon \equiv \ln \Lambda$, the elastic and viscous stresses for a taut fiber are taken as

$$\hat{\sigma}_e(\Lambda_t) = K \ln \Lambda_t, \quad \hat{\sigma}_v(\Lambda_t, \dot{\Lambda}_t) = \eta \frac{D}{Dt}(\ln \Lambda_t) = \frac{\eta \dot{\Lambda}_t}{\Lambda_t}, \tag{11}$$

where D/Dt denotes the material time derivative, K is the *elastic stiffness* and η is the *coefficient of viscosity*. Alternatively, one could assume that the reported strain is the engineering strain, $\varepsilon_0 \equiv \Lambda - 1$, and accordingly that

$$\hat{\sigma}_e(\Lambda_t) = K_0(\Lambda_t - 1), \quad \hat{\sigma}_v(\Lambda_t, \dot{\Lambda}_t) = \eta_0 \dot{\Lambda}_t, \tag{12}$$

where K_0 and η_0 are constants.

The maximum strain in the data used below, from Danto and Woo (1993) and Pioletti et al. (1999), is less than 10%. At this strain, the relative difference between the two interpretations of the reported strain is ~0.5%. That is, if $\varepsilon = 0.1$ then $\varepsilon_0 = 0.1052$ and if $\varepsilon_0 = 0.1$ then $\varepsilon = 0.0953$. While this difference is significant, it likely falls within the margin of experimental error for the data considered here. Nonetheless, it would be beneficial if experimentally determined strains (and stresses) are more clearly defined when reported (Danto and Woo, 1993). The logarithmic strain is assumed here.

3. Model implementation

In order to evaluate the capability of the proposed model, published tensile test data obtained at various strain rates will be used. To this end, an homogenous axisymmetric deformation is considered. Moreover, ACL is assumed to have a perfectly collagenous parallel-fibered structure and the recruitment process is governed by a modified Weibull distribution.

3.1. Isochoric, axisymmetric deformation

In order to compare the model with the uniaxial loading experiments reported by Danto and Woo (1993) and Pioletti et al., 1999, the isochoric deformation is assumed to be axisymmetric with a deformation gradient of the form

$$\boldsymbol{F} = \lambda^{-\frac{1}{2}}\boldsymbol{e}_r\boldsymbol{E}_R + \lambda^{-\frac{1}{2}}\boldsymbol{e}_\theta\boldsymbol{E}_\Theta + \lambda\boldsymbol{e}_z\boldsymbol{E}_Z, \tag{13}$$

where the axial stretch $\lambda(t)$ is a function of time t that satisfies $\lambda(0) = 1$ (Ogden, 1997). The orthonormal bases $\{\boldsymbol{E}_R, \boldsymbol{E}_\Theta, \boldsymbol{E}_Z\}$ and $\{\boldsymbol{e}_r, e_\theta, \boldsymbol{e}_z\}$ are defined such that $\boldsymbol{E}_Z$ and $\boldsymbol{e}_z$ are unit vectors parallel to the direction of loading in the reference and current configurations, respectively. The corresponding right Cauchy–Green deformation tensor is thus given by

$$\boldsymbol{C} = \lambda^{-1}\boldsymbol{E}_R\boldsymbol{E}_R + \lambda^{-1}\boldsymbol{E}_\Theta\boldsymbol{E}_\Theta + \lambda^2\boldsymbol{E}_Z\boldsymbol{E}_Z. \tag{14}$$

3.2. Collagen fiber orientation and crimp

In the reference configuration, the mean axial directions of the collagen fibers are assumed to be aligned along the direction of loading, $\boldsymbol{E}_Z$, so that the probability density for fiber orientation is $R(\boldsymbol{m}) = \delta(\boldsymbol{m} - \boldsymbol{E}_Z)$, where δ is the Dirac delta function. This assumption is forced by the dearth of information about the initial orientation distribution of collagen fibers in the ACL.

It then follows from (4), (8), (13), and (14) that the non-zero components of the first Piola–Kirchhoff stress are given by

$$P_{rR} = P_{\theta\Theta} = -p\lambda^{\frac{1}{2}}, \quad P_{zZ} = -p\lambda^{-1} + \sigma_{\mathrm{e}}(\lambda) + \sigma_{\mathrm{v}}(\lambda, \dot{\lambda}). \tag{15}$$

The traction-free boundary condition on the lateral surface of the test specimen thus implies that the indeterminate pressure term must vanish, $p = 0$. The recruitment model, (9) and (10), and the assumed form of the fiber response (11) then give

$$P_{zZ} = \int_1^\lambda g(\lambda_{\mathrm{s}})\left[K\ln\frac{\lambda}{\lambda_{\mathrm{s}}} + \eta\frac{\mathrm{D}}{\mathrm{D}t}\left(\ln\frac{\lambda}{\lambda_{\mathrm{s}}}\right)\right]\mathrm{d}\lambda_{\mathrm{s}} \tag{16}$$

as the only non-zero component of stress.

The crimp probability density is taken to be a modified Weibull function of the following form:

$$g(\lambda) = \alpha\beta^{-\alpha}\lambda^{-1}(\ln\lambda)^{\alpha-1}\mathrm{e}^{-(\ln\lambda/\beta)^\alpha}, \tag{17}$$

where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. This probability density function is one-sided, with $g(1) = g(\infty) = 0$ and satisfies the normality condition $\int_1^\infty g(\lambda)\mathrm{d}\lambda = 1$. The corresponding cumulative probability function is

$$G(\lambda) = \int_1^\lambda g(\lambda_{\mathrm{s}})\mathrm{d}\lambda_{\mathrm{s}} = 1 - \mathrm{e}^{-(\ln\lambda/\beta)^\alpha}. \tag{18}$$

The change of variable $\lambda_s = e^{\varepsilon_s}$ in (16) and (17) then yields

$$P_{zZ} = \int_0^{\varepsilon} \alpha\beta^{-\alpha}\varepsilon_s^{\alpha-1} e^{-(\varepsilon_s/\beta)^{\alpha}} [K(\varepsilon - \varepsilon_s) + \eta\dot{\varepsilon}] d\varepsilon_s. \tag{19}$$

This relation gives the nominal axial stress P_{zZ} as a function of the logarithmic axial strain and strain rate, ε and $\dot{\varepsilon}$, and the four material parameters K, η, α, and β, which are estimated by curve fitting data from tensile tests.

4. Results

Because of the complexities involved in conducting high strain rate experiments on collagenous tissue, there are few studies on the stress–strain relationship at such strain rates in the biomechanical literature. Published experimental stress–strain data from rabbit and bovine ACL-bone complexes (Danto and Woo, 1993; Pioletti et al., 1999) have been used to assess the proposed model. In these studies the effects of strain rate on the mechanical response of the ligamentous tissue have been investigated. Several factors such as species, age, and, most importantly, testing methodologies, have contributed to differences in the experimental findings on the ACL-bone complex rheological behavior.

Danto and Woo (1993) have tested the medial portion of rabbit ACL-bone complex by performing tensile tests at three strain rates: 0.016%/s, 1.68%/s and 381%/s. In their study, they have not observed a significant difference in the ACL mechanical properties between the strain rates of 0.016%/s and 1.68%/s. Hence, in order to determine the parameter values in the material law, (19) has been fitted using the stress–strain data at 1.68%/s and 381%/s strain rates. By implementing the Levenberg–Marquardt non-linear least-squares algorithm and by constraining the parameters to be positive, the best-fit parameters have been found to be $\alpha = 1.5$, $\beta = 0.038$, $K = 840$ MPa, and $\eta = 5.1$ MPa/s. Fig. 1 illustrates the goodness of the fit to the experimental data ($R^2 = 0.97$) together with the corresponding cumulative probability $\tilde{G}(\varepsilon) \equiv G(e^{\varepsilon}) = 1 - e^{-(\varepsilon/\beta)^{\alpha}}$. The model predicts a stiffening of the toe region with increasing strain rate. The cumulative probability shows the fraction of the fibers recruited for various values of strain. For instance, it can be observed that 90% of collagen fibers are recruited at 6.6% strain.

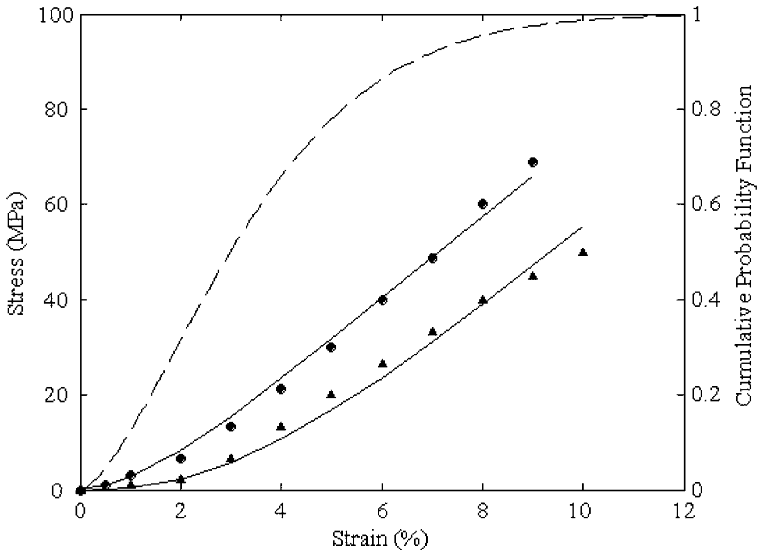

Fig. 1. Stress–strain experimental data from Danto and Woo (1993) at (▲) 1.68%/s strain rate and (●) 381%/s strain rate with (–) model and (– –) cumulative probability function evaluated at best-fitting parameters.

Pioletti et al. (1999) have conducted tensile tests at several strain rates (0.1%/s, 1%/s, 5%/s, 10%/s, 20%/s, 30%/s, 40%/s) on bovine ACL-bone complexes. They have demonstrated that the strain rate affects mainly the toe region of the stress–strain relationship while leaving almost unaffected the tangent modulus of the linear region. An attempt to fit the data of Pioletti et al. with the constitutive Eq. (19) has been made. Eq. (19) fits the experimental stress–strain curves very well with $R^2 > 0.99$ (see Fig. 2), although a unique set of

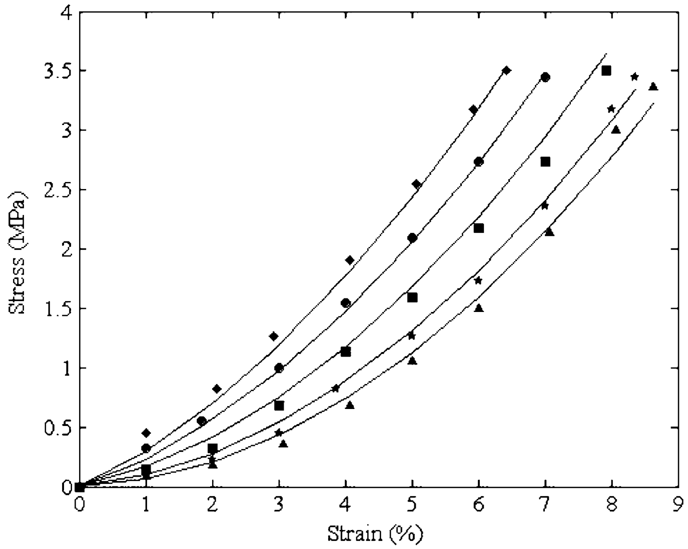

Fig. 2. Experimental data from Pioletti et al. (1999) and theoretical stress–strain curves for one ACL specimen at (▲) 5%/s, (★) 10%/s, (■) 20%/s, (●) 30%/s, (◆) 40%/s strain rates.

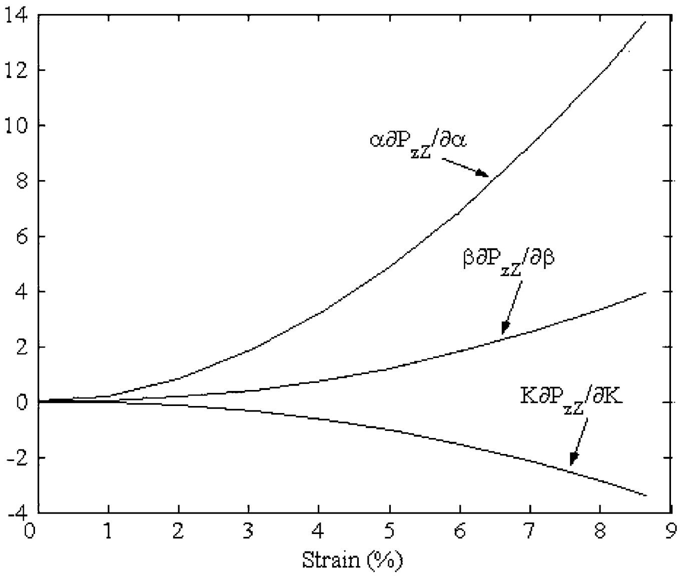

Fig. 3. Linear dependence of the sensitivity coefficients.

fitting parameters could not be determined because the identifiability criterion is not satisfied (Beck and Arnold, 1977; Belkoff and Haut, 1991). According to this criterion, the sensitivity coefficients of the model, $K\partial P_{zZ}/\partial K$, $\eta\partial P_{zZ}/\partial \eta$, $\alpha\partial P_{zZ}/\partial\alpha$, and $\beta\partial P_{zZ}/\partial\beta$, must be linearly independent in the neighborhood of the minimum sum of squares function, over the range of measurements. To illustrate the criterion, the sensitivity coefficients of the model, which have been obtained by setting $\eta = 0$ and by fitting the stress–strain data at 5%/s strain rate, are depicted in Fig. 3. The fitting parameters cannot be uniquely determined since $K\partial P_{zZ}/\partial K + \beta\partial P_{zZ}/\partial\beta$ approaches to zero and, hence, $K\partial P_{zZ}/\partial K$ and $\beta\partial P_{zZ}/\partial\beta$ are approximately linearly dependent.

5. Discussion

A constitutive law for ACL is formulated by modifying the structural theory proposed by Lanir (1979) and Lanir (1983) to include a description of strain rate-dependent effects. The non-linear material response, the anisotropy, the finite deformation, and the incompressibility of ligamentous tissue are taken into account. The model has the merit of being structurally based, with the material parameters directly associated with the tissue's main constituents.

The newly proposed model has been shown to be capable of characterizing the typical mechanical properties of the ACL with only four parameters. The initially wavy collagen fibers in the ligament become straight when subjected to a load and the tissue's overall stress–strain relationship stiffens exponentially. Consequently, the taut collagen fibers sustain the load and, hence, they are responsible for the high modulus response. The novelty of the model is in capturing, in the toe region, an increase in the mechanical properties of the ligament with strain rate. No differences in the slopes of the linear region of the stress–strain curves are predicted. These results are in accordance with the observations made by Pioletti et al. (1999) for bovine ACL.

A good fit of the model with the experimental results by Danto and Woo (1993) was attained. The estimated parameters α and β defined a distribution which describes the sequential straightening of collagen fibers as the ACL-bone complex elongates. The one-sided probability density function was selected in order to exclude the unrealistic possibility that collagen fibers straighten out in compression. In particular, the results indicated that the transition from the toe region to the linear region of the stress–strain curve occurs with a smaller percentage of fibers recruited when the strain rate is faster. It is worth noting that recent studies have established that the Weibull probability function with a non-zero location parameter can be employed to determine the reference length of the ligament (Hurschler et al., 2003). Therefore, the assumption of an initial slack configuration could be removed by introducing an additional parameter into the model.

The collagen fiber elastic modulus was found to have a value, $K = 840$ MPa, which is comparable with values reported in the biomechanical literature (Belkoff and Haut, 1991). It should be noted that, for both the current model and those cited in Belkoff and Haut (1991), the stiffness K is actually underestimated. This is because, in the model formulations, collagen fibers are assumed to occupy the whole cross sectional area of the ACL while the other ligamentous constituents have been neglected. By the same argument, the value for η is also underestimated.

When considering the experimental data from Pioletti et al. (1999), it was not possible to identify a unique set of material constants by implementing the Levenberg–Marquardt non-linear data fitting procedure and by constraining the parameters to be positive. The identifiability criterion utilizing sensitivity coefficients was invoked (Beck and Arnold, 1977; Belkoff and Haut, 1991). Indeed, the sensitivity coefficients were found to be linearly dependent. This emphasizes the need for experiments designed to accurately evaluate the parameters in the constitutive model. It is speculated that the non-uniqueness is caused by the absence of a prolonged linear region in the experimental stress–strain curves.

The proposed constitutive relation has been tested under the assumption that all collagen fibers are perfectly parallel in the ligament. Although collagen fibers are mainly oriented in the physiological loading direction of the ligament, they are not perfectly parallel to this direction. Nevertheless, this assumption can be removed when quantification of the initial fiber orientation is obtainable. Sacks (2000) showed that information on collagen fiber organization in planar tissues can be gained by using a small angle light scattering technique (SALS) and, subsequently, incorporated into structural models. For ligaments, aligned serial histological sections could be used to collect SALS data. By integrating these data from each section, complete information on the collagen fiber orientation in the ligaments could be possibly gained (Sacks and Xiaotong, 1997). Moreover, because histological studies clarifying the role of each component in the ligaments are still not available, the matrix and elastin fiber contributions together with the interactions among the various constituents of the ligament are not included in the model.

The collagen fiber behavior was modeled using a Kelvin–Voigt element, which is characterized by a linear dependence on the strain rate. It must be noted that the strain rate-dependent effects are considered to be due solely to the collagen fibers. This assumption is supported by Sasaki and Odajima (1996). In their study, it was speculated that the modulus of the collagen molecule increases with strain rate. Nevertheless, it may be possible that strain rate effects are due to intermolecular cross-linking or a fluid-like matrix. Experimental multiaxial data need to be used to accurately test the model's capability to predict the mechanical response of the ACL under various loading conditions. Tensile tests are not sufficient alone to fully characterize the mechanics of the tissue. However, strain measurements at high strain rates over the entire ligament, which are required to establish the biomechanics of the ACL, have not been performed yet.

The present study is unique in that it is the first time that a *structural* constitutive model has been formulated to reproduce the strain rate sensitivity of ACL revealed in experimental investigations. It is believed that the formulation of a reliable constitutive equation in conjunction with appropriate experimental works can lead to a better understanding of the mechanisms of ACL injury.

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